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Elastic scattering of electrons and positrons by sodium atoms

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The eikonal approximation has been used to investigate $e^{\pm}$–Na elastic scattering in the energy region $E = 1$ to 500 eV. In addition to the static potential, the effect of polarisation potential which plays a keyrole in the low and intermediate energies has also been included explicitly. Our results for the total cross-section of e^{-}–Na scattering in the energy region $3 \lesssim E \lesssim 10$ eV are in close agreement with the recent experimental results and in the energy region $10 \lesssim E \lesssim 500$ eV, the present results are very close to first Born approximation. There is a discrepancy between the present results and the available experimental findings in the energy region 10 to 50 eV. The results for e^{+}–Na scattering have also been given.

1. INTRODUCTION

The theoretical study on electron-alkali atom scattering is mostly centred round the adiabatic potential (polarised orbitals) and the close coupling approaches. Garret (1965) has applied an adiabatic model to calculate electron-lithium and electron-sodium elastic cross sections. In the light of recent experimental findings (Kasdan *et al* 1973) the cross-sections obtained are found to be very large. Karule (1965) and Karule & Peterkop (1965) have used 2-state close-coupling method to lithium, sodium, potassium and cesium. The calculations are restricted to the low energy range upto 5 eV. The most recent 4-state calculation was performed by Moores & Norcross (1972) for sodium and by Norcross for sodium and lithium (1971). These calculations are made in the range 0–5 eV. The results of the close coupling approximation are in very good agreement with the recent experimental findings. In the high energy region, there are very few calculations for the electron-sodium scattering cross section. Apart from the first Born approximation, Glauber approximation has been used by Mathur *et al* (1973) and Walters (1973) for sodium atoms. Their results do not agree with the experimental findings.

In the present paper, we have applied an eikonal approximation to calculate the elastic cross-sections of the scattering of electrons and positrons by sodium atom. This method has already been applied with success by Saha *et al* (1973) to hydrogen and helium atoms and to lithium atoms by Sarkar *et al* (1973). In this approximation the effect of polarisation potential is taken into account in addition to the static potential, the exchange effect being however neglected.

2. Theory

The target nucleus is considered to be infinitely heavy and the origin of the co-ordinate system is placed at the position of the nucleus. Let $\boldsymbol{b}$ be the impact parameter vector relative to the origin. In the eikonal approximation the scattering amplitude is given by

$$F(\theta) = \frac{k}{2\pi i} \int d^2b \exp(i\boldsymbol{q}.\boldsymbol{b})[\exp(-i\chi(\boldsymbol{b})-1], \quad \ldots \quad (1)$$

where $\boldsymbol{q}$ is the momentum transfer vector and is written as $\boldsymbol{q} = \boldsymbol{k}-\boldsymbol{k}_0$, where $\boldsymbol{k}_0$ and $\boldsymbol{k}$ represent respectively the incident and the final momentum. The phase shift function $\chi(\boldsymbol{b})$ corresponding to the impact parameter (b) may be expressed as

$$\chi(\boldsymbol{b}) = \frac{1}{\hbar v} \int_{-\infty}^{\infty} V(\boldsymbol{r})dz, \quad \ldots \quad (2)$$

where v is the velocity and $\boldsymbol{r}$ denotes the position vector of the incident particle and is given by

$$\boldsymbol{r} = \boldsymbol{b}+\hat{k}z.$$

Here $\chi(\boldsymbol{b})$ represents the combined effect of the static potential $V_s(r)$ as well as the polarisation potential $V_p(r)$. Substituting $V_s(r)$ and $V_p(r)$ in eq. (2) we have

$$\chi(\boldsymbol{b}) = (I_s+I_p)/\hbar v \quad \ldots \quad (3)$$

with

$$I_s = \int_{-\infty}^{+\infty} V_s(\boldsymbol{b}+\hat{k}z)dz \quad \ldots \quad (4a)$$

and

$$I_p = \int_{-\infty}^{+\infty} V_p(\boldsymbol{b}+\hat{k}z)dz. \quad \ldots \quad (4b)$$

The static and the polarisation potentials of Na atom are taken from Tietz (1965) and Garrett (1965) respectively. The integration in expression (4*b*) for I has been carried out numerically. The integral I_p may be written analytically as

$$I_s = Ze^2 \sum_{j=1}^{6} \alpha_j k_0(\gamma_j b), \quad \ldots \quad (5)$$

where K_0 is the Bessel function of the third kind. Eq. (1) thus can be written

$$F(\theta)=\frac{k}{i}\int_0^\infty J_0(qb)\left[\exp\left\{\frac{i}{\hbar v}(I_s+I_p)\right\}-1\right]b\,db, \quad \ldots \quad (6)$$

where J_0 is the Bessel function of the first kind. The amplitude $F(\theta)$ has been obtained by numerical integration over the impact parameter b. Standard relation has been employed to calculate the total cross section. For the case of positron—Na scattering, the sign of the static potential is to be reversed.

3. Results and Discussion

The one dimensional integration in eq. (6) has been performed numerically using Gaussian quadrature method. Special care has been taken for the oscillatory behaviour of the integrand by breaking the range of integration into several suitable parts. Results for the total cross-section in the energy range 1 to 10 eV have been displayed in figure 1 along with the corresponding the findings of Karule (1965), Karula & Peterkop (1965) and Moores & Norcross (1972). The experimental data due to Kasdan *et al* (1973) have been inserted in the same figure for comparison. Figure 2 represents present results in the energy range 10 to 500 eV along with the corresponding results due to Walter (1973) who has used the Glauber approximation. The Born results using the Tietz potential have also been given.

From figure 1 it is apparent that our values are very close to experimental findings from 4 to 10 eV. The results of Karule & Peterkop (1965) using the two-state coupling approximation and also of Moores & Norcross (1972) using

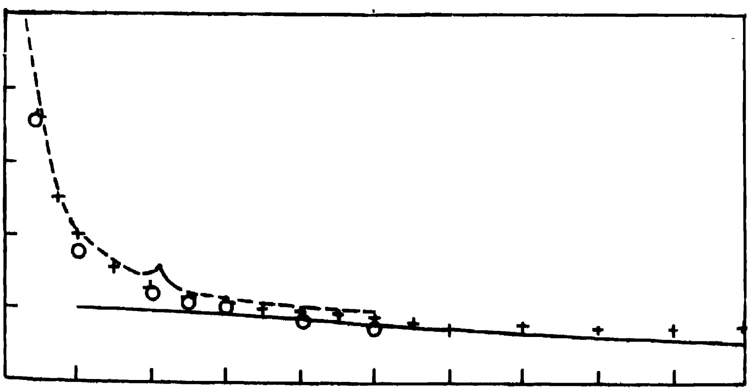

Fig. 1. Elastic electron-sodium scattering cross sections (in units of πa_0^2) present calculations; Moores & Noorcross; O, Kerule, Kerule & Peterkop and + +, Experimental points (Kasdan *et al*).

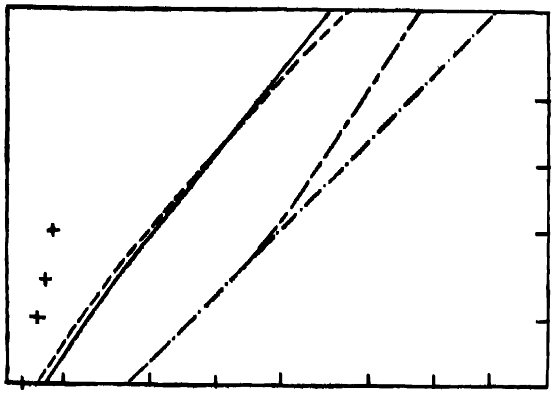

Fig. 2. Elastic electron-sodium scattering cross sections (in units of πa_0^2) present results; — First Born; ---- Walters with core potential; -·-·- Walters without core potential.

4-state calculation are in close agreement with our present results in the energy range 3 to 5 eV. From 1 to 3 eV our curve lies below the experimental points and the results of other theoretical calculations. This disagreement may be due to the fact that we have ignored the exchange effect in our approach. A similar discrepancy has also been noticed in the case of e^-–H, e^-–He (Saha *et al* 1973, 1974) and e^-–Li (Sarkar *et al* 1973) scattering using the same approximation. From 10 to 500 eV the present results for the total cross section are very close to Born results. The results of Walter (1973) differ from the present as well as Born calculations appreciably even upto 500 eV. The experimental values are given upto 50 eV. The present results which are in close agreement with the observed values in the energy range 3 to 10 eV differ appreciably in the energy range 10 to 50 eV. At 50 eV the observed value is around 2.5 greater than the present result whereas it is eight times greater than the value obtained by Walters (1973) using Glauber approximation. The differences between the theoretical results and the experimental findings in the energy region 10 to 50 eV may be due to the choice of the effective potentials. Therefore, we think that more sophisticated calculations are required to determine the exact nature of the cross section in the high energy region.

In table 1, we have given the total elastic cross section for the e±-Na scattering from 1 to 500 eV. The results obtained in the case of positrons are always less than the corresponding values of electron. To our knowledge no work has been may be performed by taking positron as incident particle. Therefore we could not say anything definite about this results. Future experimental measurement or theoretical calculations will justify our results.

Table 1. Total cross-sections for elastic e⁻–Na scattering (in units of πa_0^2)

Electron energy E(eV)	Positron	Electron	Electron energy E(eV)	Positron	Electron
1.0	160.387	113.672	12.0	28.675	53.055
2.0	115.369	110.047	15.0	23.309	45.555
3.0	89.384	102.412	20.0	18.083	37.552
4.0	72.482	95.086	50.0	8.436	18.726
5.0	60.701	87.595	100.0	4.787	10.778
6.0	52.233	80.821	200.0	2.745	6.389
7.0	46.001	74.784	500.0	1.447	3.126
8.0	41.122	69.126			
10.0	33.748	59.948			

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